Agents with left and right dominant hemispheres and quantum statistics

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We present a multiagent model illustrating the emergence of two different quantum statistics, Bose-Einstein and Fermi-Dirac, in a friendly population of individuals with the right-brain dominance and in a competitive population of individuals with the left-brain hemisphere dominance, correspondingly. Doing so, we adduce the arguments that Lefebvre's "algebra of conscience" can be used in a natural way to describe decision-making strategies of agents simulating people with different brain dominance. One can suggest that the emergence of the two principal statistical distributions is able to illustrate different types of society organization and also to be used in order to simulate market phenomena and psychic disorders, when a switching of hemisphere dominance is involved.

I. INTRODUCTION

A study of social and economical processes can be based on use of the *multiagent models* [1]. In such models each agent must imitate a human being who has to survive in the environment using proper decision-making strategies and interacting with other agents. Many of these models use neural networks for an agent to be represented [2]. However, to describe market phenomena, one should make such agents be rather different: a homogeneous agent population does not simulate a market behavior plausibly. As a result, heterogeneity is often artificially included in an agent population. However, this procedure is often far-fetched and in reality has no relation to the data of a real brain functioning and to human cognitive abilities. In short, the neural network models are too primitive to represent the brain functioning, and it is necessary to search the agent's model of a higher level.

Here we present a multiagent model which in a natural way includes the diversity of the decision-making strategies by the agents. The diversity is caused by the dominance of the different human brain hemispheres. This model has some predecessors.

The first one is connected to remarkable insight into market phenomena described by Williams [3]: market losers (90% of the participants) appear to be the persons with a strong dominance of the left brain hemisphere. These persons are governed by fear and avidity (the feelings considered, e.g., by Peters [4] to be necessary ingredients of the market economy), and they obviously use their logical and mathematical capabilities (attached to their left brain hemisphere) for the decision making. These left brainers can be described with a specific model of the world, as they consider it as a vertical hierarchical structure with a permanent competition, occasional slumps, and difficult climbs to upper levels of the social hierarchy. On the other hand, market winners seem to be more right-brain hemispheric. These right brainers do not feel any fear, rely upon their intuition, and consider the environment to be a friendly place for cooperation and forming horizontal relations. A natural question we want to discuss below is as follows: can we simulate this difference in the world picture inherent to people with different brain dominance using simple mathematical models? We suggest that it will be natural for this purpose to use a multiagent model in which one can simulate the left- and right-dominant agents. But how can these two types of the agents be described?

Here, the second predecessor can be useful. In the *algebra of conscience* developed by Lefebvre [5], the possibility of the real existence of two types of ethical systems only is reasoned. We suppose that this dichotomy can arise due both to the brain asymmetry and to left or right brain dominance.

We will also demonstrate that Lefebvre's elegant formalism can be naturally used to develop a model of agents with left- and right-brain dominance. Moreover, we will also adduce arguments that two reasonable decision-making strategies only arise in populations in which agents try to preserve their *physical* and *mental* resources. Obviously, the most interesting feature of the model is the one that societies of pure left- and right-brain dominance agents, with competitive and friendly relations correspondingly, are described using the famous quantum distributions: the Fermi-Dirac and Bose-Einstein ones. We propose that this fact is in accordance with our goal: to simulate the existence or nonexistence of the hierarchy (verticality) in the society and in the world picture of the people with the different brain dominance.

We will not be surprised that such famous distributions arise in our classic multiagent model. There are different systems, both quantum and classic, the state of equilibrium of which is described with quantum statistical distributions.

For example, Evans has found the Bose-Einstein condensation (BEC) while solving the heterogeneous transport problem (particle hopping) [6]. Bianconi and Barabasi [7] demonstrated that the Bose-Einstein statistics describes the growing Internet (this network grows constantly by means of adding and removing new sites and links). Staliunas [8] adduced arguments that the BEC can arise in classic systems far from thermal equilibrium due to the system coherent dynamics or due to the equivalent autocatalytic dynamics in a system momentum space. An essential condition for the Bose-Einstein distribution to appear in this case is the fact that random particle migration through momentum space is dependent on the state occupation degree within this space.

This is a typical situation for many nonlinear systems. Therefore the quantum nature of the system is not an essential ingredient for the BEC. In addition, Bianconi has found recently that a growing Cayley tree with a different number of nodes and with a thermal noise is described by the Fermi-Dirac statistics [9]. Earlier, Derrida and Lebowitz [10] found both the Fermi-Dirac, and Bose-Einstein distributions when studying fully asymmetric exclusion processes at a ring containing N sites and p particles. We will demonstrate that quantum statistical distributions also describe populations of the agents living in the cellular world model, which is introduced in the next section.

II. CELLULAR WORLD MODEL

Let us suppose that the *world* consists of *n cells* which in general can contain an arbitrary number of *agents*, $x^{(\alpha)}$, $\alpha = 1, \ldots, N$.

Every agent has two kinds of *resources*: *physical* and *mental* ones, which are characterized by real values $p^{(\alpha)} \geq 0$ and $m^{(\alpha)} \geq 0$.

The agent $x^{(\alpha)} = \{p^{(\alpha)}, m^{(\alpha)}\}$ *dies* if any of its resources takes zero value. So, to survive, any agent should maintain its physical and mental resources to be positive at any time $t \geq 0 : p^{(\alpha)}(t) \geq 0$ and $m^{(\alpha)}(t) \geq 0$.

Preventing *both resources* from vanishing is, in general, *a contradictory problem.*

Let us suppose that every agent should use some physical resource $\gamma \Delta t (\gamma > 0)$ for the time interval Δt to hold its physical structure. This process is accompanied by *an unconditioned* decrease of the agent's physical resource.

Fortunately, each agent can *consume* some amount of the physical resource (food), $h \Delta t$ $(h > 0)$, which appears at random in the world cells. Doing so, the agent has to change the cell, if this food appears in another cell (it has to go to the latter cell). If the food appears in the agent's cell, this agent can consume it without changing the cell.

We suggest that the same portion of the physical resource (food) appears in different cells with a different frequency f_i, $i = 1, \ldots, n$, reflecting the attractiveness of the given cell for the agent.

Suppose now that if an agent *changes its cell* to consume the food, *its mental resource decreases by one.* We interpret this situation as if the agent consumes its mentality to solve a problem of physical survival. We also suggest that the agent cannot increase or compensate its mental resource by no means. Surely, if the food appears in the agent's cell, it can consume it without changing its mental resource.[1]

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[1] It is necessary to further clarify our concepts of the world cells and mental surviving. One should not consider the world cells as the cells of some physical space. They do not have neighbor or far cells, so no metrics are introduced. Each of the world cells can be characterized by a set of parameters, such as (musician, south). In this case the fact whether the food appears in a Milan orchestra or in a Barcelona orchestra is not principle for the agent being in this cell. The agent can change its real geographical position, but will stay in the same world cell. On the contrary, if the food appears in a Stockholm orchestra, the agent should change its cell, because the

We interpret an appearance of food in the cell being free of a specific agent as the *environment proposal to change the agent's mentality* or which is the same as to pay a unit of mental resource for the food.

Let the Boolean variable a denote this proposal, and $a = 0$ if the environment offers to change the cell.

The appearance of the unit of physical resource in the cell occupied by given agent can be considered as a proposal for the agent *to preserve its mentality* and to consume the food *for free.*

Let $a = 1$, if the environment offers the agent to keep its cell.

Let us suppose that every agent can accept or reject such a proposal and that its decision is a Boolean function of a: $\psi(a)$. Let $\psi(a) = 0$ mean that the agent decides to change its cell and to occupy another one expending its mental resource but consuming the food which (unfortunately) is in the other cell. Similarly, $\psi(a) = 1$ means that the agent decides to stay in its old cell. The last decision is accompanied by saving its mental resource and by decreasing its physical resource (if $a = 0$) or by increasing of physical resource for free, if the latter fortunately appears in the same agent's cell (if $a = 1$).

A. Interaction-free model

Further we will introduce an interaction between agents, but first consider an interaction-free model.

Let us demonstrate that *two different strategies* for the noninteracting agents to survive exist in the cellular world described above.

We have already suggested that one can describe agent's decision by the Boolean function of one variable:

$$\psi = \psi(a). \tag{1}$$

There are four different functions of this kind, and we will consider all of them.

(i) If $\psi(a) \equiv 0$, then the agent changes its cell every time, when the food appears, even if it appears in the cell, where the agent is already located. This strategy is *absolutely unreasonable*, because it results in inevitable decrease of the mental resource down to the fatal zero value—i.e., to the agent's *mental death.*

(ii) If $\psi(a) = \bar{a}$, then the agent always acts against the environment proposal: it changes its cell if the food appears just in it, and reserves its cell, if the food appears in another cell. Obviously, such an agent gradually loses its physical resource up to it's *physical death.*

One should note that in both the above-mentioned cases, a new cell, where the agent tries to move while the environment does not demand such a change, is not specified. So the agent's behavior in such cases looks like a random walk, and

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cell with the food is now (musician, north). It will be also necessary for the agent to change the cell in order to consume the food, if it is demanded to become a cowboy, or to go to the East. It may appear that the concept of the world cells is rather subjective. But we suggest that all the agents have the same conception of the world cell division, so this cellular structure can be considered as an objective one.

its dynamics is partly *stochastic*.

Now let us consider two *reasonable strategies*.

(i) The first one is described by the Boolean function

$$\psi(a) = 1. \tag{2}$$

The agent keeps its mental resource regardless of the cell in which the food appears. As this food can occasionally appear in the agent's cell, the latter one has also a chance to survive physically, if such a lucky event happens frequently enough.

(ii) The second reasonable strategy is described by the function

$$\psi(a) = a. \tag{3}$$

Using this strategy the agent always follows the proposals of the environment increasing its physical resource by food consuming. The agent also has a chance to keep its mental resource, when the environment does not demand to change the cell of its location.

We will give some interpretation of the two reasonable strategies. Doing so, it is convenient to represent the corresponding Boolean functions in exponential form:

$$\psi_R(a) = 1 \equiv a^a, \tag{4}$$

$$\psi_L(a) = a \equiv a^{\bar{a}}, \tag{5}$$

where

$$a^c = a + \bar{c} = c \rightarrow a \tag{6}$$

is a logical implication. We will name these strategies *right-brain* and *left-brain strategies*, correspondingly. We will also name the agents following these strategies *right (left) brain agents*. One can present some preliminary arguments in favor of this interpretation. There is some experimental evidence that the right hemisphere is not able to create logical negations: all logical operations are functions of the left hemisphere. Thus, a decision making for a right-brain agent in the case $a = 0$ can be interpreted as follows.

A right-brain agent *thinks* that *follows the environmental proposal* to change the cell and decreases its mental resource. This possibility to be lost while changing the world cell *horrifies the agent* and it *denies* this environment proposal.

On the other hand, a left-brain agent is able to create a mental image corresponding to the *logical inversion of the environment proposal*. So its decision making can be described as follows.

A left-brain agent *thinks* that *denies the environmental proposal* to consume the food in any cell. This possibility to slip a chance of increasing its physical resource *horrifies it*, and it *accepts* the environment proposal.

We will further see that other arguments in favor of this interpretation of the two reasonable strategies exist.[2]

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[2] One of such arguments was presented by Rotenberg and Arshavsky. They suggested that "...in its most general form the difference between the two strategies of thinking is reduced to opposite modes of organizing the contextual connections between

B. Right-brain strategy

Let *the agent-environment interaction*, which consists in the proposal to an agent h units of the physical resource, have the characteristic time scale τ. Thus, the probability for the agent not to receive such a proposal decreases as $e^{-t/\tau}$.

The dynamics of the population consisting of right-brain agents only is very simple. These agents do not change their cells and do not change their mental resources as well. If the number of agents in the cell i is equal to N_i, then

$$N_i(t) \equiv N_i(0), \tag{7}$$

$$m^{(\alpha)}(t) \equiv m^{(\alpha)}(0). \tag{8}$$

Their physical resource, $p^{(\alpha)}(t)$, however, changes with time. Suppose that for τ time an agent consumes at an average $\gamma\tau$ units of the physical resource. Then,

$$p^{(\alpha)}(t + \Delta t) = p^{(\alpha)}(t) - \gamma\frac{\Delta t}{\tau} + f_i h\frac{\Delta t}{\tau}, \quad \alpha \in C_i, \tag{9}$$

where $\alpha \in C_i$ means that the αth agent occupies cell i. Assuming that $\Delta t \rightarrow 0$, we obtain

$$\frac{d}{dt}p^{(\alpha)} = -\frac{1}{\tau}(\gamma - hf_i), \quad \alpha \in C_i. \tag{10}$$

It follows from Eq. (10) that

$$p^{(\alpha)}(t) = p^{(\alpha)}(0) - \frac{1}{\tau}(\gamma - hf_i)t, \quad \alpha \in C_i. \tag{11}$$

Thus, a right-brain agent will survive in those cells for which $hf_i \geq \gamma$. For the cells with $hf_i < \gamma$, their physical life will have the duration

$$T_{phys}^{(\alpha)} = \frac{\tau p^{(\alpha)}(0)}{\gamma - hf_i}, \quad \alpha \in C_i. \tag{12}$$

Hence the right-brain strategy is absolutely passive, and an agent's survival only depends on the parameters of the envi-

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elements of information. 'Left-hemisphere' mode of thinking so organizes any sign material (whether symbolic or iconic) as to create a strictly ordered and unambiguously understood context. Its formation requires an active choice, out of the real and potential connections between the multiform objects and phenomena of a few definite connections, which would not create internal contradictions (!) and would facilitate an ordered analysis...In contrast, the function of 'right-hemispheric,' 'image' thinking is a simultaneous capture of an infinite number of connections and the formation due to this capture of an integral but *ambiguous context*. In such a context, the whole is not determined by its components since all specific features of the whole are determined only by interconnections between these parts. On the contrary, any concrete element of such a context bears a determining stamp of the whole. A new experience is incorporated in this holistic picture of the world. Individual facets of images interact with each other on many semantic planes simultaneously. Examples of such contextual connections are the connections between images in sleep dreams or in work of art. The advantages of this strategy of thinking manifest themselves only when the information itself is complex, internally contradictory and basically irreducible to an unambiguous context." [11].

ronment and on the chance to occupy an initially lucky cell.

C. Left-brain strategy: Gibbs distribution

Left-brain agents change their cells to consume the food offered them. Hence the averaged particle numbers in a cell ($\langle N_i \rangle$) becomes a time function $\langle N_i(t) \rangle$. Surely, we have to use average values, because the food proposal is a random process. Within the time interval $[t, t+\Delta t]$ the average N_i values will increase due to the arrival of the agents accepting the proposal to consume the food in the ith cell and will decrease because some agents that initially occupied cell i receive the food proposal in other cells. The balance relation is as follows:

$$\langle N_i(t + \Delta t) \rangle = \langle N_i(t) \rangle + \sum_{j \neq i} f_i \left(\frac{\Delta t}{\tau} \right) \langle N_j(t) \rangle - \sum_{j \neq i} f_j \left(\frac{\Delta t}{\tau} \right) \langle N_i(t) \rangle. \tag{13}$$

At the limit $\Delta t \to 0$ we obtain

$$\frac{d}{dt} \langle N_i(t) \rangle = -\frac{1}{\tau} \langle N_i(t) \rangle + \frac{f_i N}{\tau}. \tag{14}$$

The solution has the form

$$\langle N_i(t) \rangle = (\langle N_i(0) \rangle - f_i N) e^{-t/\tau} + N f_i. \tag{15}$$

It is evident that the distribution of the mean occupation tends to the distribution of the food proposal frequencies:

$$\lim_{t \to \infty} \langle N_i(t) \rangle = N f_i. \tag{16}$$

According to [4], we introduce the cell energy.

$$\epsilon_i = -\theta \ln f_i, \tag{17}$$

where the parameter θ characterizes the *temperature* of the environment.

Then the equilibrium distribution (16) takes the form of *the Gibbs' distribution*:

$$\lim_{t \to \infty} \langle N_i(t) \rangle = N e^{-\epsilon_i/\theta}. \tag{18}$$

To obtain the averaged period of the left-brain agents survival, one should remember that they gradually lose their mental resource changing their cells.

By definition, the average mental resource for all agent population is equal to

$$\langle m(t) \rangle = \frac{1}{N} \sum_{i=1}^{n} \sum_{\alpha \in C_i} m_i^{(\alpha)}(t), \tag{19}$$

where C_i is a set of the indexes of the agents occuping the ith cell. The average mental resource of the ith cell agents is equal to

$$\langle m_i(t) \rangle = \frac{1}{\langle N_i(t) \rangle} \sum_{\alpha \in C_i} m_i^{(\alpha)}(t). \tag{20}$$

Let us write down the balance relation for the integral mental resource of the ith cell agents, which reflects the out-

flow of the agents with a previous resource value and the inflow of the agents from other cells with resources reducing by one:

$$\langle N_i(t + \Delta t) \rangle \langle m_i(t + \Delta t) \rangle = \langle N_i(t) \rangle \langle m_i(t) \rangle - \sum_{j \neq i} f_j \left(\frac{\Delta t}{\tau} \right) \langle N_i(t) \rangle$$

$$\times \langle m_i(t) \rangle + \sum_{j \neq i} f_i \left(\frac{\Delta t}{\tau} \right) \langle N_j(t) \rangle$$

$$\times (\langle m_i(t) \rangle - 1) \tag{21}$$

or

$$\langle N_i(t + \Delta t) \rangle \langle m_i(t + \Delta t) \rangle = \langle N_i(t) \rangle \langle m_i(t) \rangle \left(1 - \frac{\Delta t}{\tau} (1 - f_i) \right)$$

$$+ \sum_{j \neq i} f_i \left(\frac{\Delta t}{\tau} \right) \langle N_j(t) \rangle (\langle m_i(t) \rangle - 1). \tag{22}$$

Taking the limit $\Delta \to 0$ we obtain

$$\frac{d}{dt} \langle N_i \rangle \langle m_i \rangle = -\frac{1}{\tau} (1 - f_i) \langle N_i \rangle \langle m_i \rangle + \frac{f_i}{\tau} \left(\sum_{j \neq i} \langle N_j \rangle \langle m_j \rangle - \sum_{j \neq i} \langle N_j \rangle \right). \tag{23}$$

As

$$\sum_{j \neq i} \langle N_j \rangle = N - \langle N_i \rangle \tag{24}$$

and

$$\sum_{j \neq i} \langle N_j \rangle \langle m_j \rangle = \langle Nm \rangle - \langle N_i \rangle \langle m_i \rangle = N \langle m \rangle - \langle N_i \rangle \langle m_i \rangle, \tag{25}$$

after simple algebra, we obtain

$$\frac{d}{dt} \langle N_i \rangle \langle m_i \rangle = -\frac{1}{\tau} \langle N_i \rangle \langle m_i \rangle + \frac{N f_i}{\tau} (\langle m \rangle - 1) + \frac{\langle N_i \rangle f_i}{\tau}. \tag{26}$$

Summing Eq. (26) over $i = 1, \ldots, n$ and taking into account that

$$\sum_{i=1}^{n} \langle N_i \rangle \langle m_i \rangle = N \langle m \rangle, \tag{27}$$

we receive

$$\frac{d}{dt} \langle m \rangle = -\frac{1}{N\tau} \sum_{i=1}^{n} (1 - f_i) \langle N_i(t) \rangle. \tag{28}$$

Now, using the explicit form of $\langle N_i(t) \rangle$ one can integrate Eq. (28):

$$\langle m(t) \rangle = \langle m(0) \rangle - \frac{t}{\tau} \sum_{i=1}^{n} f_i (1 - f_i) - \frac{1}{N} \sum_{i=1}^{n} [\langle N_i(0) \rangle - f_i N](1 - f_i)$$

$$\times (1 - e^{-t/\tau}). \tag{29}$$

From this expression we can derive a transcendental equation for the time T_m of the mental survival by setting $\langle m(T_m) \rangle = 0$:
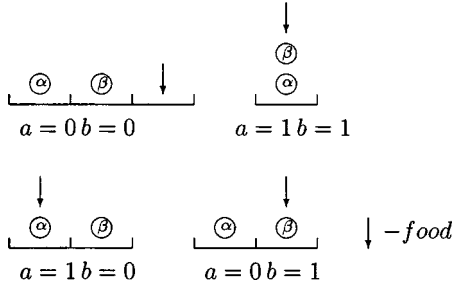
$$\langle m(T_m) \rangle = 0: \tag{30}$$

FIG. 1. Four different cases of food proposals and the respective environment's proposals, a and b, to agents α and β, correspondingly

$$T_m = \frac{\tau\langle m(0)\rangle}{\sum_{i=1}^{n} f_i(1-f_i)} - \frac{\tau}{N\sum_{i=1}^{n} f_i(1-f_i)}$$

$$\times \sum_{i=1}^{n} [\langle N_i(0)\rangle - f_i N](1-f_i)(1-e^{-T_m/\tau}). \quad (31)$$

If $\langle m(0)\rangle$ is big enough, we have

$$T_m \cong \frac{\tau\langle m(0)\rangle}{\sum_{i=1}^{n} f_i(1-f_i)}. \quad (32)$$

It follows from Eq. (32) that if the distribution of food proposals f tends to be concentrated in one cell, $f_i \to \delta_{ik}$, then $T_m \to \infty$. It means that if the food is offered in a single cell, then the left-brain agents immediately occupy it and can keep their mental resource constant forever. One should note that the total survival will be only guaranteed if the physical survival will be provided by the food proposals.

III. MENTAL INTERACTION OF AGENTS

We can further develop our model assuming that an agent can take into account the proposal to another agent before it makes the decision. One can interpret it as if agent α "interacts mentally" with other agents. We suppose that this interaction is a pairwise one, so that each agent can take into account the situation with only one *arbitrary* agent, including itself. Specifically, we suggest that if food is offered to agent α, *this agent thinks that it is also offered to agent β*—see Fig. 1. In accordance with Lefebvre, we also suggest that an agent can consider two types of relations with another agent: friendly and competitive ones.

Now, the decision of agent α depends both on the environment proposal a to agent α and also on its proposal b to agent β *from the point of view of agent α that the same unit of the physical resource is offered to agent β*. Now an intention of agent α becomes a function of two variables:

$$\psi = \psi(a,b). \quad (33)$$

We suggest that the function describing the decision of a right-brain agent coincides with the one describing the intention of the person which is attributed by Lefebvre to the *second ethical system* [5]:

$$\psi_R = a^{a*b}, \quad (34)$$

TABLE I. The decisions of the right brain agent taking into account the proposal to the friend (the third column) and to the enemy (the fourth column).

a	b	a^{a+b}	$a^{a\cdot b}$
0	0	1	1
0	1	0	1
1	0	1	1
1	1	1	1

where $* = +$ if a *right-brain agent* α believes that agent β is its *friend* and $* = \cdot$ if α believes that agent β is its *enemy*.

The decision (intention) of the left-brain agent will be determined in accordance with the noninteractive case by logical negation of the exponent index. Thus,

$$\psi_L = a^{\overline{a*b}}. \quad (35)$$

Here, in accordance with Lefebvre's definition related to the agents belonging to the *first ethical system* (which is identified with the left-hemisphere dominance in the presented model), we suppose that $* = \cdot$ if the *left-brain agent* α believes that agent β is its *friend* and $* = +$ if it believes that agent β is its *enemy*.

One should note that if the intention form of the right-hemisphere dominance agent in our model *coincides* with the intention of the individual belonging to the second ethical system introduced by Lefebvre, the intention form of the left-brain agent *differs* from the one belonging to the first ethical system. This difference is expressed in the logical negation of the term in the exponent (35).

Let us consider the intentions of the two types of agents as a function of two variables a and b.

For the right-brain agent α which considers β as its friend, we obtain the function values presented in the third column of Table I. Let us focus on the second row of this column. One can conclude that the right-brain agent accepts the environment proposal to consume the food in a new cell diminishing the mental resource (as a matter of fact acting as a left-brain agent) only if the environment does not demand to do this of its friend (with which agent α interacts mentally).

In other words, the right-brain agent changes its cell and moves to another one (where the food is offered), if its friend with which agent α interacts mentally is already in this new cell. Let us name this strategy of the right-brain agent as *moving to a friend*.

For the right-brain agent which considers agent β as an enemy, we obtain the decisions presented in the fourth column of Table I.

We see that the enemies *do not influence* the intentions of the right-brain agent at all. One can conclude that the right-brain agents in our model only take into account the situations with their friends.

For the left-brain agent which interacts mentally with a friend (i.e., with a friend from its point of view), we obtain (the third column of Table II) that friends of a left-brain agent do not influence its decisions. On the contrary, for the

TABLE II. Decisions of the left-brain agent taking into account the proposal to the friend (the third column) and to the enemy (the fourth column).

a	b	$a^{\overline{a \cdot b}}$	$a^{\overline{a+b}}$
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	1

left-brain agent which takes into account the situation with an enemy, the decisions are presented in the fourth column of Table II. Again, focusing on the second row of this column, we conclude that the left-brain agent acts as the right-brain agent, not following the environment proposal to consume the food in an other cell, unless the environment demands of the enemy to change its cell (so that the food is offered just in the cell which occupies a random enemy, with which agent α interacts mentally). In the other words, a left-brain agent does not move to a new cell for food if the enemy is already there. Let us name this strategy of the left-brain agent *not join an enemy*. One can conclude that the left-brain agents only take into account the situation with their enemies.

One should note that the decision of a left-brain agent can be presented in a simpler form, which does not demand Lefebvre's use of different operations for describing friendly and competitive relations in different ethical systems. Using "+" and "·" for them (as for a right-brain agent), one can replace the function $a^{\overline{a \cdot b}}$ with an identical function $a^{\overline{a}+\overline{b}}$ and function $a^{\overline{a+b}}$ with an identical function $a^{\overline{a}\cdot\overline{b}}$. These new forms can be easily interpreted as those in which a left-brain agent thinks that it rejects the environment proposal together with the agent it interacts mentally.

It is important to note that the described rules of the agent's mental interactions can be only confirmed using this model for a description of real-world phenomena. The main confirmation of them is that these rules correspond to the rules developed by Lefebvre for the second ethical system (evidenced experimentally [5]) and to the hypothesis that the left hemisphere only can perform logical operations.

A. Asymmetry of an agent's self-interaction

We should also consider the situation when an agent interacts *with itself* while decision making. Recall that, e.g., the function $\psi(a,b) = a^{a+b}$ describes the situation when right-brain agent α *imagines* a situation of friendly relations with agent β (mentally interacts with it). If agent α chooses $\beta = \alpha$, it can imagine a situation of *self-interaction*. As long as it is difficult to imagine that an agent is friendly or competitive to itself, it is natural to consider just the agent's *reflexion* as such a self-interaction.

For a right-brain agent one can express this reflexion as

$$\psi = a^{a^a}. \tag{36}$$

This expression means that the right-brain agent thinks itself to accept the environment proposal. It is evident that for such a reflexive (self-interacting) agent,

$$\psi = a + \overline{a^a} = a + \overline{a + \overline{a}} = a + \overline{1} \equiv a. \tag{37}$$

Therefore, this agent acts as a left-brain agent. So *the self-interaction of a right-brain agent converts it into a left-brain agent.*

Imagine for a moment that it can also imagine itself to be a left-brain agent (though it should not be permitted in our model, where a right-brain agent cannot perform a logical negation). Then,

$$\psi = a^{a^{\overline{a}}} = a + \overline{a^{\overline{a}}} = a + \overline{a} \equiv 1. \tag{38}$$

In this case it really acts as a right-brain agent. On the contrary, a reflexive left-brain agent acts as a nonreflexive left-brain agent regardless of its mental constructions (whether it imagines itself as a left-brain agent or a right-brain one). Indeed, irrespective of the choice $-a$ or $\overline{a}$, we obtain

$$\psi = a^{\overline{a}^a} = a + \overline{\overline{a} + \overline{a}} = a \tag{39}$$

and also

$$\psi = a^{\overline{a}^{\overline{a}}} = a + \overline{\overline{a}^{\overline{a}}} = a + \overline{a + \overline{a}} \equiv a. \tag{40}$$

We make a conclusion that an evident asymmetry between reflexive right-brain agents and left-brain ones exists in our model. A reflexive right-brain agent acts as a left-brain agent, but a reflexive left-brain agent remains a left-brain agent. Below we will demonstrate that this transformation of the behavior of a reflexive right-brain agent leads to a Bose-Einstein distribution in the population of friendly right-brain agents.

B. General case of agent interaction

In the first part of the paper we considered the simplest cases of populations dynamics for noninteracting agents with different hemisphere dominance. For mentally interacting agents, analogous consideration (concerning, e.g., the agent survival) is more complicated and we intend to consider this problem in our further studies. Here we only note that an agent's survival depends on the delicate interplay of the agent cell distribution and food proposal distribution. It also strongly depends on the specific structure of the interagent relations (friendly or competitive). Qualitatively, the more agents are considered by a given right brainer as friends, the more its mobility in the cell world and the more chances to avoid physical death. On the other hand, the more agents are considered by a given left brainer as enemies, the more probability to reject food proposals and the more chances to save its mental resource. In general, for arbitrary interagent relations, one can study a multiagent system mainly using a computer simulation and not an analytical approach.

One can see from the following simple examples how mental interactions complicate the model properties (see Fig. 2). In the first case, two right brainers, which are friendly to each other, occupy two world cells (e.g., initially one agent in a cell) [see Fig. 2(a)]. If the food is offered in the both cells, e.g., with equal probability, then behavior of these agents will be qualitatively similar to a noninteracting left brainer. So these interacting right brainers can die because of
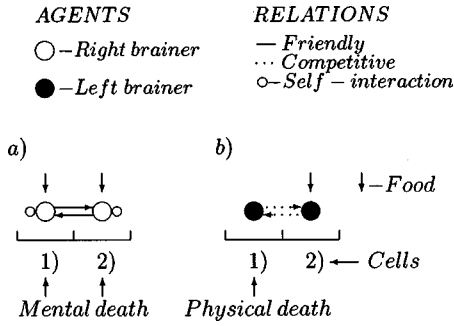
FIG. 2. (a) Two friendly self-interacted right brainers (both agents consider each other as friends) living in a two-cell world express *qualitatively* the behavior of noninteracting left brainers if food is presented in both cells. These agents can die due to lack of mental resource. (b) The left brainer occupying the first cell having no food supply can die due to the lack of physical resource (as noninteracting right brainers) if it is in competitive relations with the other left brainer occupying the second cell which has the food supply. Note that in the last case we exclude the agents self-interaction, so the agent from the first cell has no chance to go to the second cell.

expiring their *mental resources*. In a sense their behavior seems to be unreasonable, because food proposals can give them in some cases a possibility to survive physically without changing the cell. In the second case we consider two left brainers. If the agent initially occupying the first cell is in a competitive relation with the other left brainer [Fig. 2(b)] occupying the second cell and the food is only offered in the latter cell, then behavior of the first left brainer without self-interaction will be identical to a right brainer in the interaction-free model. For example, the agent occupying the first cell can die due to the lack of the *physical resource*. Note that the influence of the self-interaction (reflexion) which really gives the agents a minimal mobility decreases with the increase of the agents number in the population, N. Despite the complexity of the general model of mentally interacted agents it is of interest to consider its limiting cases leading us to the quantum statistics.

C. Right-brain strategy: Bose-Einstein distribution

Let us consider the community of the right hemisphere dominant agents which are friendly to each other.[3] We have already considered the case of the noninteracting right-brain agents and showed that they occupy their initial cells at any time t. The appearance of the interaction permits the right-brain agent to change its cell if (i) it interacts mentally with its friend occupying the cell in which the food appears (*moving to a friend*) and (ii) it interacts mentally with itself—this reflexive interaction converts its behavior into the left-brain-like one and permits to change its original cell.

Let us suppose that agent α occupies cell j, but the food is offered in cell i. Let agent α interact mentally with a random agent (including itself). The probability that it will choose

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[3]We have already noted that this type of relations is typical for the right-brain agents.

the agent occupying just cell i is $\langle N_i \rangle / N$, while the probability to choose itself is equal to $1/N$. So the total probability for agent α to move to cell i is

$$p_{j \to i} = (\langle N_i \rangle + 1)/N. \tag{41}$$

It is just the probability for bosons to occupy state i. Hence its application leads to the Bose-Einstein distribution of the agents in the cell space.

Indeed, as the probability for the of food to appear in cell i is f_i, then the probability for agent α to occupy a new cell i within time interval $[t, t + \Delta t]$ is proportional to $[\langle N_i(t) \rangle + 1] f_i$. After normalization it can be written as

$$p_{j \to i} = \frac{[\langle N_i(t) \rangle + 1] f_i}{N}. \tag{42}$$

As follows from the *detailed balance principle*, at equilibrium the rates of exchange between two cells i and j are equal to each other,

$$\langle N_j \rangle (\langle N_i \rangle + 1) f_i = \langle N_i \rangle (\langle N_j \rangle + 1) f_j, \tag{43}$$

or, taking into account Eq. (17),

$$\frac{\langle N_i \rangle}{\langle N_i \rangle + 1} e^{\epsilon_i / \theta} = \frac{\langle N_j \rangle}{\langle N_j \rangle + 1} e^{\epsilon_i / \theta}. \tag{44}$$

As the last relation is satisfied for every i and j, the expression on the left-hand side is constant $e^{\mu/\theta}$, where μ is the chemical potential. So,

$$\frac{\langle N_i \rangle}{\langle N_i \rangle + 1} e^{\epsilon_i / \theta} = e^{\mu/\theta}. \tag{45}$$

It follows directly from the last equation that

$$\langle N_i(\epsilon_i) \rangle = \frac{1}{e^{(\epsilon_i - \mu)/\theta} - 1}. \tag{46}$$

D. Left-brain strategy: Fermi-Dirac distribution

Now let us consider the community of left-brain agents which are competitive to each other. According to Williams [3], just the *competitive* relations are typical to left-brain persons. On the other hand, friendly relations, as we mentioned above, do not influence the decisions of the left-brain agents. Besides, we have already considered the case of the noninteracting left-brain agents and showed that at equilibrium they reach Gibbs distribution. The appearance of a mental interaction permits a left-brain agent to hold its cell if it interacts mentally with its enemy already occupying the cell with the food (no join an enemy).

Let us suppose that agent α occupies cell j and the food is offered in cell i. Let agent α randomly choose an agent (including itself) for the mental interaction. The probability that it will choose an agent not occupying cell i is $(N - \langle N_i \rangle)/N$. Therefore, the probability for the agent α to move to cell i is also

$$p_{j \to i} = \frac{N - \langle N_i \rangle}{N}. \tag{47}$$

Note that agent α can choose itself as a partner for the mental interaction. But this cannot change its intention to move to cell i, because unlike the case of a right-brain agent, the self-interaction does not change the behavior of the reflexive left-brain agent.

Again, as the probability of the food appearance in cell i is f_i, the probability for agent α to occupy a new cell i within time interval $[t, t+\Delta t]$ is equal to

$$p_{j \to i} = \frac{[N - \langle N_i(t) \rangle] f_i}{N}. \tag{48}$$

Using the principle of detailed balance again, we take into account that at equilibrium the rate of exchange between two cells i and j is equal to

$$\langle N_j \rangle (N - \langle N_i \rangle) f_i = \langle N_i \rangle (N - \langle N_j \rangle) f_j \tag{49}$$

or

$$\frac{\langle N_i \rangle}{N - \langle N_i \rangle} e^{\epsilon_i/\theta} = \frac{\langle N_j \rangle}{N - \langle N_j \rangle} e^{\epsilon_j/\theta}. \tag{50}$$

As it was before, seeing that the last relation is satisfied for every i and j, the expression on the left-hand side is a constant $e^{\mu/\theta}$. So,

$$\frac{\langle N_i \rangle}{N - \langle N_i \rangle} e^{\epsilon_i/\theta} = e^{\mu/\theta}. \tag{51}$$

From the last equation it follows that

$$\langle N_i(\epsilon_i) \rangle = \frac{N}{e^{(\epsilon_i - \mu)/\theta} + 1}. \tag{52}$$

So at equilibrium the community of the left-brain agents being competitive with each other is described by the Fermi-Dirac distribution. If thermodynamic equilibrium can be realized within the time interval when no agent dies due to exhausting either physical or mental resources, then the value of the chemical potential μ can be calculated using normalization both for the community of the left brainers and of the previously considered right brainers:

$$\sum_{i=1}^{n} \langle N_i(\epsilon_i) \rangle = N. \tag{53}$$

This relation reflects the conservation of the agent number.

For the right-brain agents described by the Bose-Einstein distribution we obtain a relation from which one can find the value of the chemical potential:

$$\sum_{i=1}^{n} \frac{1}{e^{(\epsilon_i - \mu)/\theta} - 1} = N. \tag{54}$$

It is evident that its value is, in general, temperature dependent.

For the left-brain agents obeying Fermi-Dirac statistics, we similarly obtain

$$\sum_{i=1}^{n} \frac{1}{e^{(\epsilon_i - \mu)/\theta} + 1} = 1. \tag{55}$$

The principle of detailed balance says that if $\langle N_1 \rangle$ and $\langle N_2 \rangle$ are the average number of states marked with 1 and 2, then at equilibrium the number of transitions from 1 to 2 must be equal to those from 2 to 1. This principle is stronger than the equilibrium condition itself and deeply related to the microscopic reversibility and Onsager reciprocity. It can be used not only when deriving the Fermi-Dirac distribution for *fermions* and the Bose-Einstein distribution for *bosons*, but also to receive the intermediate quantum statistical distributions for *anyons* [12]. It is remarkable that the statistical distribution for anyons can be derived without using the spin-statistics theorem.

IV. HEMISPHERE SWITCHING AND INTERMEDIATE QUANTUM STATISTICS

Some further possible applications of this model should be outlined. It is clear that by approaching the critical regimes when the physical or mental resource expires, an agent *can change* its strategy of survival by changing the dominant hemisphere.[4] Therefore, the hemisphere switching dynamics can be naturally incorporated in the model. It is well known that disturbances of hemisphere dominance cycles are considered by some authors as a source of different mental disorders.

For example, Pettigrew and Miller [13] supposed that a decrease of the binocular competition rate—this competition is just connected with the hemisphere dominance switching—is an indicator of the bipolar disorder [13].[5]

Hence, the dynamics of the hemisphere switching can be used to simulate, e.g., the manic-depressive syndrome hypothetically caused by interactions of agents in a population. It clears the way for the social conditions to be taken into account in the progress of mental disorders.

From this point of view it would be interesting to study the most interesting general case of a population consisting of the agents with different hemispheric dominance and to find its equilibrium states. It seems that in this case more general forms of the quantum statistics would be relevant.

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[4]Note that in Lefebvre's *algebra of conscience* a change of agent relations—from friendly to competitive and *vice versa*—is considered as a way to increase its *ethical status*. At the same time, the *ethical system* of the agent does not change. In our approach it is more fruitful to consider the *strategy switching* while holding the nature of interagent relations.

[5]The evidence that interhemispheric switching is connected to the deep depression and that this disorder can be initiated or *reduced* by electric stimulation of a half of the brain is presented in [14]. Recently Dodson considerably reduced manic symptoms of a patient by pouring cold water into the left ear [15]. In general, the left hemisphere is overactive in the case of mania, compared with the overactivity of the right hemisphere in the case of depression. Since thermal vestibular activation is effective for one hemisphere, the mania could be cured by left thermal activation which would enhance the right-side activity and thereby would reduce the mania.

For example, as was shown by Huang [16], if a transmutation of bosons and fermions is allowed (in our case it corresponds exactly to the hemisphere dominance switching), then a system with bosons and fermions has the function of anyons statistical distribution.

Intermediate statistics types can be found not only in the systems of quantum quasi-particles [17]. For example, Bianconi presented a case of the nonhomogeneous growing *complex network* with different features of nodes showing a mixed quantum statistics [18].

One should consider the crucial problem of the transmutation mechanism incorporation, and it seems to be extremely important, as it allows determining the fractions of the left- and right- hemisphere-dominant agents in the multiagent model with hemisphere switching.

One should mention another possible direction of the model development. Despite the emergence of quantumlike statistics in the just-presented model, we suppose that the agents act in the strictly classic way: *either* as right-hemisphere-dominant persons *or* as left-hemisphere-dominant ones (with the possibility of dominance switching). So every fact of the quantumlike statistical behavior of an agent population will not imply the quantum nature of the agents themselves. Nevertheless, it is possible to extend this *classic model* to the *quantum domain* supposing that the agents can be in superposition:

$$\psi = \beta|\text{right}\rangle + \gamma|\text{left}\rangle, \qquad (56)$$

where β and γ are complex amplitudes of the event, when an agent acts as a right- or a left-brain-dominant person, correspondingly. Such a model can have some relation to the *ambiguous statistics* considered by Medvedev [19]. In this type of statistics all particles have unknown type. This can be due, e.g., to particle-type oscillations, when particles are bosons in given period, but then transmute to fermions and vice versa.[6] During pairwise interaction a particle recognizes the type of the other one (and vice versa) and interacts with it according to its revealing type.

If the probability for the particle to be recognized as a boson (fermion) is p_b (p_f), then these particles will obey the anyon statistics, which can be derived using a deformed commutation relation

$$a_i a_j^\dagger - q a_j^\dagger a_i = \delta_{ij}, \qquad (57)$$

where $q=(p_b-p_f)/(p_b+p_f)$. Of course, it is important to find out whether such generalizations of the presented model have *some* new *experimentally checkable* features. But this question is out of the scope of this article.

V. CONCLUSION

We have demonstrated how one can develop the multiagent model describing populations of agents with different brain hemisphere dominance and reasoned that these populations obey the known famous quantum statistics and potentially are described by intermediate quantum statistics. This confirms the possibility to use quantum statistics when studying social and economical phenomena.

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[6]So brain hemisphere dominance oscillations can be naturally described.

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